D.R.G.Government Degree College

Tadepalligudem, West Godavari District

CBCS 2020-21 REVISED SYLLABUS

I SEMESTER
PHYSICS PAPER-I

Mechanics, Waves and Oscillations

STUDY MATERIAL
(ENGLISH MEDIUM)

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B.Sc. PHYSICS SYLLABUS UNDER CBCS
For Mathematics Combinations
[2020-21 Batch onwards]
I Year B.Sc.-Physics: I Semester
Course I: MECHANICS, WAVES AND OSCILLATIONS

Work load: 60 hrs per Semester **4 hrs/week**

UNIT-I:
1. Mechanics of Particles (5 hrs)
Review of Newton's Laws of Motion, Motion of variable mass system, Motion of a rocket, Multistage rocket, Concept of impact parameter, scattering cross-section, Rutherford scattering-Derivation.

2. Mechanics of Rigid bodies (7 hrs)
Rigid body, rotational kinematic relations, Equation of motion for a rotating body, Angular momentum and Moment of inertia tensor, Euler equations, Precession of a spinning top, Gyroscope, Precession of atom and nucleus in magnetic field, Precession of the equinoxes

Unit-II:
3. Motion in a Central Force Field (12hrs)
Central forces, definition and examples, characteristics of central forces, conservative nature of central forces, Equation of motion under a central force, Kepler's laws of planetary motion- Proofs, Motion of satellites, Basic idea of Global Positioning System (GPS), weightlessness, Physiological effects of astronauts

UNIT-III:
4. Relativistic Mechanics (12hrs)
Introduction to relativity, Frames of reference, Galilean transformations, absolute frames, Michelson-Morley experiment, negative result, Postulates of Special theory of relativity, Lorentz transformation, time dilation, length contraction, variation of mass with velocity, Einstein's mass-energy relation

Unit-IV:
5. Undamped, Damped and Forced oscillations: (07 hrs)
Simple harmonic oscillator and solution of the differential equation, Damped harmonic oscillator, Forced harmonic oscillator – Their differential equations and solutions, Resonance, Logarithmic decrement, Relaxation time and Quality factor.
6. Coupled oscillations: (05 hrs)
Coupled oscillators-Introduction, Two coupled oscillators, Normal coordinates and Normal modes- N-coupled oscillators and wave equation

Unit-V:
7. Vibrating Strings: (07 hrs)
Transverse wave propagation along a stretched string, General solution of wave equation and its significance, Modes of vibration of stretched string clamped at ends, Overtones and Harmonics,Melde's strings.
8. Ultrasonics: (05 hrs)
Ultrasonics, General Properties of ultrasonic waves, Production of ultrasonics by piezoelectric and magnetostriction methods, Detection of ultrasonics, Applications of ultrasonic waves, SONAR

Course outcomes:
On successful completion of this course, the students will be able to:
- ✓ *Understand Newton's laws of motion and motion of variable mass system and its application to rocket motion and the concepts of impact parameter, scattering cross section.*
- ✓ *Apply the rotational kinematic relations, the principle and working of gyroscope and it's applications and the precessional motion of a freely rotating symmetric top.*
- ✓ *Comprehend the general characteristics of central forces and the application of Kepler's laws to describe the motion of planets and satellite in circular orbit through the study of law of Gravitation.*
- ✓ *Understand postulates of Special theory of relativity and its consequences such as length contraction, time dilation, relativistic mass and mass-energy equivalence.*
- ✓ *Examine phenomena of simple harmonic motion and the distinction between undamped, damped and forced oscillations and the concepts of resonance and quality factor with reference to damped harmonic oscillator.*
- ✓ *Appreciate the formulation of the problem of coupled oscillations and solve them to obtain normal modes of oscillation and their frequencies in simple mechanical systems.*
- ✓ *Figure out the formation of harmonics and overtones in a stretched string and acquire the knowledge on Ultrasonic waves, their production and detection and their applications in different fields.*

UNIT-I

1. Mechanics of particles

Newton's laws of motion

Newton's first law:

Every body continues to be in a state of rest or uniform motion unless it is acted by an external force.

Newton's second law:

The net external force acting on a body is directly proportional to the rate of change of momentum.

$$F = \frac{dP}{dt} = \frac{d}{dt}(mv) = m\frac{dv}{dt} = ma$$

$$F = ma$$

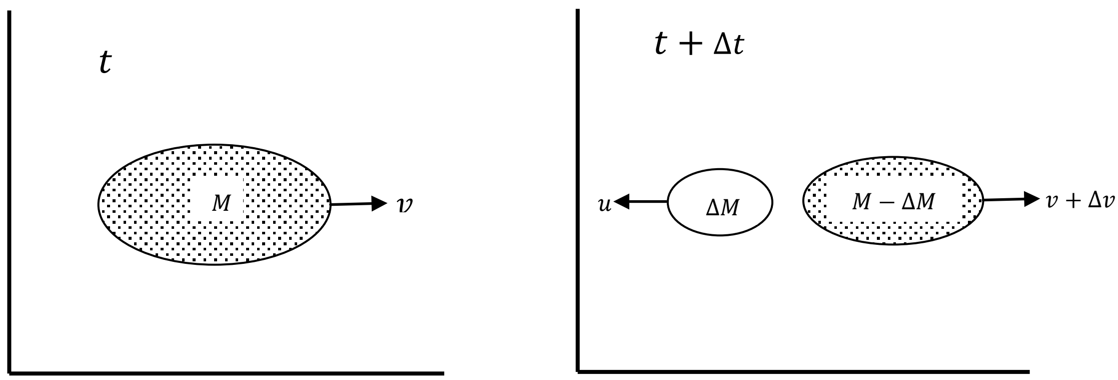
Newton's third law:

Every action has an equal and opposite reaction.

$$F_{12} = -F_{21}$$

Equation of motion of a system of variable mass

If the mass of system changes with time without remaining constant, such a system is known as a system of variable mass. Motion of rocket is an example of system of variable mass. When the fuel inside the combustion chamber of a rocket is burnt, the burnt gases are ejected from the rocket in the form of a gas jet with high velocity in backward direction. As a result mass of the rocket decreases gradually and its velocity increases.



Consider a system of mass M moving with velocity v as shown in figure. After a time Δt, a mass ΔM is ejected from the system with velocity u. As a result, mass of the system is reduced to $(M - \Delta M)$and its velocity increased to$(v + \Delta v)$.

Initial momentum $P_i = Mv$

Final momentum $P_f = (M - \Delta M)(v + \Delta v) + \Delta M u$

Change in momentum $\Delta P = P_f - P_i = (M - \Delta M)(v + \Delta v) + \Delta M u - Mv$

According to Newton's second law

$$F_{ext} = \frac{dP}{dt} = \frac{\Delta P}{\Delta t} = \frac{(M - \Delta M)(v + \Delta v) + \Delta M u - Mv}{\Delta t} = \frac{Mv + M\Delta v - v\Delta M - \Delta v \Delta M + \Delta M u - Mv}{\Delta t}$$

$$= \frac{M\Delta v - v\Delta M - \Delta v \Delta M + \Delta M u}{\Delta t} = M\frac{\Delta v}{\Delta t} - v\frac{\Delta M}{\Delta t} - \Delta v\frac{\Delta M}{\Delta t} + u\frac{\Delta M}{\Delta t}$$

$$If \;\; \Delta t \to 0, then \;\; \frac{\Delta v}{\Delta t} = \frac{dv}{dt}, \;\; \frac{\Delta M}{\Delta t} = -\frac{dM}{dt}, \Delta v \approx 0$$

$$F_{ext} = M\frac{dv}{dt} + v\frac{dM}{dt} - u\frac{\Delta M}{\Delta t} = \frac{d}{dt}(Mv) - u\frac{\Delta M}{\Delta t}$$

$$\boldsymbol{F_{ext} = \frac{d}{dt}(Mv) - u\frac{\Delta M}{\Delta t}}$$

The above equation represents the equation of motion of a system of variable mass.

$$F_{ext} = M\frac{dv}{dt} + v\frac{dM}{dt} - u\frac{\Delta M}{\Delta t}$$

$$M\frac{dv}{dt} = F_{ext} + u\frac{\Delta M}{\Delta t} - v\frac{dM}{dt}$$

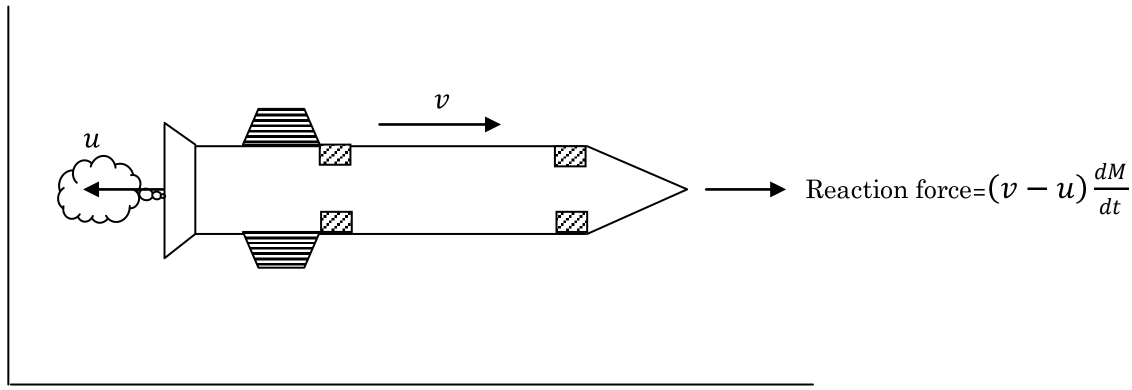
$$\boldsymbol{M\frac{dv}{dt} = F_{ext} + (u - v)\frac{dM}{dt}}$$

Reaction force or thrust acting on the rocket is gieven by

$$\boldsymbol{F_{reaction} = (u - v)\frac{dM}{dt}}$$

$$\boldsymbol{M\frac{dv}{dt} = F_{ext} + F_{reaction}}$$

Expression for final velocity of a rocket



Motion of rocket is an example of system of variable mass. When the fuel in the combustion chamber of a rocket is burnt, pressure inside the chamber increases. Hence the hot gases inside the combustion chamber are ejected from the rocket in the form of a gas jet with high velocity in backward direction through a nozzle. Hence mass of the rocket decreases gradually due to the ejected gases and its velocity increases.

Consider a rocket of mass M moving with a velocity v at time t as shown in figure. After a time Δt, fuel of mass dM is ejected from the rocket with a velocity u in the form of a gas jet. Hence the velocity of the gas jet relative to the laboratory frame of reference is $(v - u)$.

Relative velocity $v_{relative} = v - u$

Reaction force on the rocket

$$F_{reaction} = (v - u)\frac{dM}{dt}$$

External force on the rocket

$$F_{ext} = -Mg$$

Hence the resultant force on the rocket in upward direction

$$F = F_{reaction} + F_{ext}$$

$$F = (v - u)\frac{dM}{dt} - Mg$$

According to Newton's second law

$$F = \frac{dP}{dt} = \frac{d}{dt}(Mv)$$

$$\frac{d}{dt}(Mv) = (v - u)\frac{dM}{dt} - Mg$$

$$M\frac{dv}{dt} + v\frac{dM}{dt} = v\frac{dM}{dt} - u\frac{dM}{dt} - Mg$$

$$M\frac{dv}{dt} = -u\frac{dM}{dt} - Mg$$
$$\frac{dv}{dt} = -\frac{u}{M}\frac{dM}{dt} - g$$
$$dv = -u\ \frac{dM}{M} - gdt$$
Let v_0, v be the initial final velocities and M_0, M be the initial and final masses of the rocket. Integrating the above equation on both sides
$$\int_{v_0}^{v} dv = -u\ \int_{M_0}^{M}\frac{dM}{M} - g\ \int_{0}^{t} dt$$
$$(v)_{v_0}^{v} = -u(\log M)_{M_0}^{M} - g(t)_0^t$$
$$v - v_0 = -u(\log M - \log M_0) - gt$$
$$v - v_0 = -u\log\frac{M}{M_0} - gt$$
$$v - v_0 = u\log\frac{M_0}{M} - gt$$

$$\boldsymbol{v = v_0 + u\log\frac{M_0}{M} - gt}$$

The above expression represents the final velocity of the rocket.

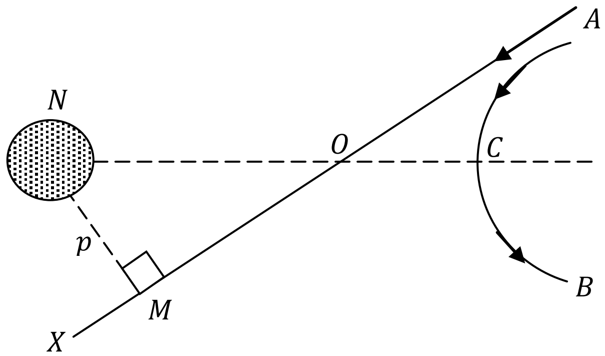
Case(i):

Ignoring gravity,$g \approx 0$,
$$\boldsymbol{v = v_0 + u\log\frac{M_0}{M}}$$

Case(ii):

If the initial velocity of the rocket is zeroi.e$\boldsymbol{v_0} = 0$
$$\boldsymbol{v = u\log\frac{M_0}{M}}$$

Impact Parameter

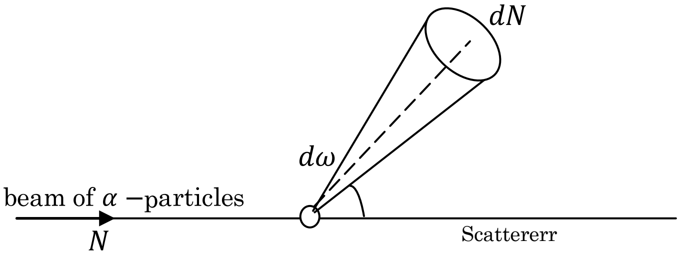


Consider an alpha particle of mass m and charge $+2e$ moving towards a nucleus of charge $+Ze$ in AX direction. Alpha particle follows a hyperbolic path ACB instead of a straight path AX due to coulomb's repulsion of the nucleus. p is the perpendicular distance from nucleus $N$to the initial direction of alpha particle. This is known as Impact parameter. Hence Impact parameter can be defined as follows.

- Impact parameter $(p)$is defined as the perpendicular distance from the nucleus to the initial direction of the projected alpha particle.

If Impact parameter $p = 0,$ then the collision is known as direct collision. In this case, the scattering angle $\emptyset = 0$.

Collision cross-section (or) Scattering cross-section

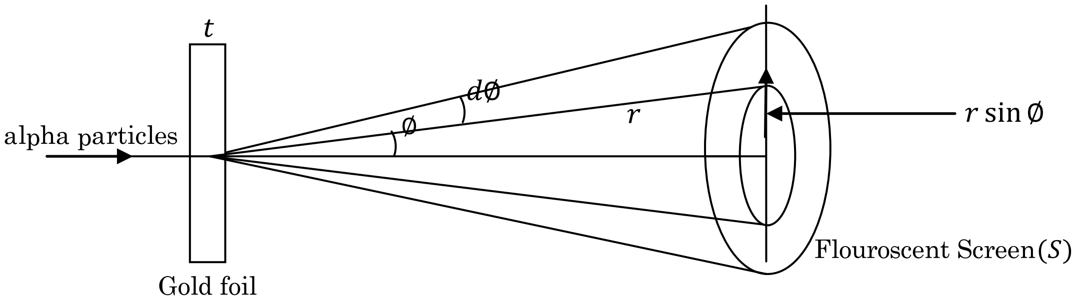


When alpha particles are incident on a thin gold foil, they are scattered in different directions. Let Nbe the incident intensity of alpha particle. Let dN be the number of alpha particles scattered in to solid angle $d\omega$. The ratio of number of alpha particles scattered in to solid angle $d\omega$ and the incident intensity is known as Impact parameter.

Scattering cross-section$(\sigma) = \frac{Number\ \ of\ alpha\ particles\ \ scattered\ \ in\ to\ sloid\ angle\ \ d\omega}{Incident\ \ intensity}$

$$\sigma = \frac{dN}{N}$$

Rutherford's Scattering Cross-section



Consider a narrow beam of alpha particles incident normally on a gold foil as shown in figure. Alpha particles are scattered in different directions due to coulomb's repulsive force of the nucleus. A fluorescent screen (S) is used to detect the scattered alpha particles. Let t be the thickness of the gold foil and N be the number of atoms per unit volume. Let Q be the number of alpha particles incident on the gold foil per unit area. Any alpha particle which comes within a distance of impact parameter $(p)$from the nucleus will be scattered through an angle Ø. Hence in order to calculate the number of alpha particles scattered through an angle Ø, let us imagine a circle of radius equal to impact parameter around each nucleus. Total area of all such circles is $\pi p^2 nt$.

- Probable number of alpha particles which can come within a distance p from the nucleus
$$= \pi p^2 ntQ.$$
- Number of alpha particles having impact parameter between p and $p + dp$
   - $= d(\pi p^2 ntQ) = 2\pi pntQdp$
- Hence the number of alpha particles having scattered through an angle between Ø and $\emptyset + d\emptyset$
$$= 2\pi pntQdp$$

Scattering cross-section$(\sigma) = \frac{Number\ \ of\ alpha\ particles\ \ scattered\ \ in\ to\ sloid\ angle\ \ d\omega}{Incident\ \ intensity}$

Solid angle between Øand $\emptyset + d\emptyset = 2\pi \sin\emptyset\, d\emptyset$

- Hence number of alpha particles scattered in to solid angle $d\omega$
$$= \sigma\, I\, d\omega = \sigma\, I\, 2\pi \sin\emptyset\, d\emptyset$$

This value should be equal to the number of alpha particles having impact parameter between p and $p + dp$.

- Number of alpha particles having impact parameter between p and $p + dp$

$$= 2\pi p\ dp$$

$$\text{Number of incident alpha particles} = 2\pi p\ dp.I$$

$$\therefore \quad \sigma\ I\ 2\pi \sin\emptyset\ d\emptyset = -2\pi p\ dp.I$$

$$\sigma = \frac{-2\pi p\ dp.I}{2\pi \sin\emptyset\ d\emptyset.I} = -\frac{p\ dp}{\sin\emptyset\ d\emptyset}$$

$$\sigma = -\frac{p\ dp}{\sin\emptyset\ d\emptyset}$$

$$p = \frac{Ze^2}{2\pi\epsilon_0 m v_0^2}\cot\frac{\emptyset}{2}$$

$$dp = \frac{Ze^2}{2\pi\epsilon_0 m v_0^2}\left(-\frac{1}{2}\operatorname{cosec}^2\frac{\emptyset}{2}d\emptyset\right)$$

$$\sigma = \frac{\left(\frac{Ze^2}{2\pi\epsilon_0 m v_0^2}\right)^2 \cot\frac{\emptyset}{2}.\frac{1}{2}\operatorname{cosec}^2\frac{\emptyset}{2}d\emptyset}{\sin\emptyset\ d\emptyset} = \frac{Z^2e^4\cot\frac{\emptyset}{2}.\operatorname{cosec}^2\frac{\emptyset}{2}}{8\pi^2\epsilon_0^2 m^2 v_0^4.2\sin\frac{\emptyset}{2}.\cos\frac{\emptyset}{2}} = \frac{Z^2e^4}{16\pi^2\epsilon_0^2 m^2 v_0^4\sin^4\frac{\emptyset}{2}}$$

$$\boldsymbol{\sigma = \frac{Z^2e^4}{16\pi^2\epsilon_0^2 m^2 v_0^4\sin^4\frac{\emptyset}{2}}}$$

This is known as Rutherford' Scattering cross-section.

Rutherford's scattering formula:

Number of alpha particles scattered through an angle between $\emptyset$and $\emptyset + d\emptyset$

$$= 2\pi pntQdp$$

Substituting the values of p and $p + dp$ in the above equation,

Number of alpha particles scattered through angle between $\emptyset$and $\emptyset + d\emptyset$

$$= 2\pi ntQ\left(\frac{Ze^2}{2\pi\epsilon_0 m v_0^2}\cot\frac{\emptyset}{2}\right)\left[\frac{Ze^2}{2\pi\epsilon_0 m v_0^2}\left(-\frac{1}{2}\operatorname{cosec}^2\frac{\emptyset}{2}d\emptyset\right)\right]$$

These particles strike the screen (S)in a circular annulus of area dA

$$dA = 2\pi r\sin\emptyset\ rd\emptyset = 2\pi r^2\sin\emptyset\ d\emptyset = 4\pi r^2\sin\frac{\emptyset}{2}\cos\frac{\emptyset}{2}\ d\emptyset$$

Number of alpha particles incident on the screen per unit area

$$N = \frac{2\pi ntQ\left(\frac{Ze^2}{2\pi\epsilon_0 m v_0^2}\cot\frac{\emptyset}{2}\right)\left[\frac{Ze^2}{2\pi\epsilon_0 m v_0^2}\left(-\frac{1}{2}\operatorname{cosec}^2\frac{\emptyset}{2}d\emptyset\right)\right]}{4\pi r^2\sin\frac{\emptyset}{2}\cos\frac{\emptyset}{2}\ d\emptyset}$$

$$\boldsymbol{N = \frac{QntZ^2e^4}{16\pi^2\epsilon_0^2 m^2 v_0^4\sin^4\frac{\emptyset}{2}}}$$

This is known as Rutherford's scattering formula.

Hence from the above equation, it is clear that the number of alpha particles scattered per unit area is

- Inversely proportional to $\boldsymbol{\sin^4\frac{\emptyset}{2}}$
- Directly proportional to the thickness of gold foil $'t'$
- Directly proportional to the square of the atomic number $'Z'$ of the scatterer
- Inversely proportional to the square of the kinetic energy of the alpha particle.

Unit-I

2. Mechanics of Rigidbodies

Euler's equations (or) Equations of motion of a rigid body

Equation of motion of a rigid body in space coordinate system

$$\vec{\tau}_{\text{space}} = \left(\frac{d\vec{L}}{dt}\right)_{\text{space}} \quad (1)$$

The rotation of a rigid body can also be studied a coordinate system fixed in the rigid body.This is known as body corrdinate system.

$$\vec{L}_{\text{body}} = (\overleftrightarrow{I}\vec{\omega})_{\text{body}} \quad (2)$$

We can transform the equations of motion of a rigid body from body cocordinate system to space coordinate system using the operator given below.

$$\left(\frac{d}{dt}\dots\right)_{\text{space}} = \left(\frac{d}{dt}\dots\right)_{\text{space}} + \vec{\omega}\times(\dots)$$

$$\left(\frac{dL}{dt}\right)_{\text{space}} = \left(\frac{dL}{dt}\right)_{\text{space}} + \vec{\omega}\times\vec{L}$$

From equations 1 and 2

$$\vec{\tau} = \frac{d\vec{L}}{dt} + \vec{\omega}\times\vec{L}$$

If the body is symmetric, the axes of rotation cooincide with the principal axis of symmetry.In this case, except the diagonal elements I_{xx}, I_{yy}, I_{zz}, the non-diagonal elements in of the inertia tensor will be zero.

$$Let I_{xx} = I_1, I_{yy} = I_2, I_{zz} = I_3$$

$$\vec{\omega}\times\vec{L} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \omega_1 & \omega_2 & \omega_3 \\ L_1 & L_2 & L_3 \end{vmatrix}$$

$$= \vec{i}(\omega_2 L_3 - \omega_3 L_2) + \vec{j}(\omega_3 L_1 - \omega_1 L_3) + \vec{k}(\omega_1 L_2 - \omega_2 L_1)$$

Hence in X direction

$$\tau_1 = \frac{dL_1}{dt} + (\omega_2 L_3 - \omega_3 L_2)$$

Since $L = I\omega$

$$\tau_1 = I_1\frac{d\omega_1}{dt} + (\omega_2 I_3 \omega_3 - \omega_3 I_2 \omega_2)$$

$$\boxed{\tau_1 = I_1\frac{d\omega_1}{dt} + (I_3 - I_2)\omega_2\omega_3}$$

Similarly in Y, Z directions

$$\boxed{\tau_2 = I_2\frac{d\omega_2}{dt} + (I_1 - I_3)\omega_1\omega_3}$$

$$\boxed{\tau_3 = I_3\frac{d\omega_3}{dt} + (I_2 - I_1)\omega_1\omega_2}$$

The above three equations are known as Euler's equations of motion of a rigid body. Expressing these equations in terms of x, y, z.

$$\tau_x = I_x\frac{d\omega_x}{dt} + (I_z - I_y)\omega_y\omega_z$$

$$\tau_y = I_y\frac{d\omega_y}{dt} + (I_x - I_z)\omega_x\omega_z$$

$$\tau_z = I_z\frac{d\omega_z}{dt} + (I_y - I_x)\omega_x\omega_y$$

Expressing these equations in symmetric form

$$\tau_x = I_x\frac{d\omega_x}{dt} - (I_y - I_z)\omega_y\omega_z$$

$$\tau_y = I_y \frac{d\omega_y}{dt} - (I_z - I_x)\omega_z\omega_x$$
$$\tau_z = I_z \frac{d\omega_z}{dt} - (I_x - I_y)\omega_x\omega_y$$

Applications of Euler's equations:

Law of conservation of energy:

Euler's equations of motion are given by

$$\tau_1 = I_1 \frac{d\omega_1}{dt} + (I_3 - I_2)\omega_2\omega_3$$
$$\tau_2 = I_2 \frac{d\omega_2}{dt} + (I_1 - I_3)\omega_1\omega_3$$
$$\tau_3 = I_3 \frac{d\omega_3}{dt} + (I_2 - I_1)\omega_1\omega_2$$

When there is no external torque acting on the rigid body $\tau = 0$

$$I_1 \frac{d\omega_1}{dt} + (I_3 - I_2)\omega_2\omega_3 = 0$$
$$I_2 \frac{d\omega_2}{dt} + (I_1 - I_3)\omega_1\omega_3 = 0$$
$$I_3 \frac{d\omega_3}{dt} + (I_2 - I_1)\omega_1\omega_2 = 0$$

Multiplying the above equations with $\omega_1, \omega_2, \omega_3$ respectively and adding we get

$$I_1 \frac{d\omega_1}{dt}\omega_1 + I_2 \frac{d\omega_2}{dt}\omega_2 + I_3 \frac{d\omega_3}{dt}\omega_3 = 0$$
$$\frac{1}{2}\frac{d}{dt}[I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2] = 0$$
$$I_1\omega_1^2 + I_2\omega_2^2 + I_1\omega_3^2 = K = \text{Rotational kinetic energy}$$
$$\frac{1}{2}\frac{d}{dt}(2K) = 0$$
$$\frac{dK}{dt} = 0$$
$$K = \text{Constant}$$

Hence the rotational kinetic energy of a rigid body remains constant in the absebse of net external torque.

Law of conservation of angular momentum:

When there is no external torque acting on the rigid body $\tau = 0$

$$I_1 \frac{d\omega_1}{dt} + (I_3 - I_2)\omega_2\omega_3 = 0$$
$$I_2 \frac{d\omega_2}{dt} + (I_1 - I_3)\omega_1\omega_3 = 0$$
$$I_3 \frac{d\omega_3}{dt} + (I_2 - I_1)\omega_1\omega_2 = 0$$

Multiplying the above equations with $I_1\omega_1, I_2\omega_2, I_3\omega_3$ respectively and adding we get

$$I_1{}^2 \frac{d\omega_1}{dt}\omega_1 + I_2{}^2 \frac{d\omega_2}{dt}\omega_2 + I_3{}^2 \frac{d\omega_3}{dt}\omega_3 = 0$$
$$\frac{1}{2}\frac{d}{dt}\left[I_1{}^2\omega_1^2 + I_2{}^2\omega_2^2 + I_3{}^2\omega_3^2\right] = 0$$
$$\frac{1}{2}\frac{d}{dt}[L^2] = 0$$
$$L\frac{dL}{dt} = 0$$
$$\frac{dL}{dt} = 0$$
$$L = \text{Constant}$$

Hence the angular momentum of a rigid body remains constant in the absense of net external torque.

Unit-II

3. Central forces

A force which always acts towards or away from a fixed point and whose magnitude depends only on the distance of the particle from the fixed point is known as a central force.

$$\text{Central force } \vec{\boldsymbol{F}} = \boldsymbol{f}(\boldsymbol{r})\hat{\boldsymbol{r}}$$

Examples:

1. Gravitational force is a central force.

Gravitational force between two objects of masses m_1, m_2 separated by a distance r is given by

$$\vec{F} = -\frac{Gm_1m_2}{r^2}\hat{r}$$

$$Let - Gm_1m_2 = C$$

$$\therefore \vec{F} = \frac{C}{r^2}\hat{r}$$

$$f(r) = \frac{C}{r^2}$$

$$\therefore f(r) \propto \frac{1}{r^2}$$

2. Electrostatic force is a central force.

Electrostatic force between two particles of charges q_1, q_2 separated by a distance r is given by

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}\hat{r}$$

$$Let \frac{q_1q_2}{4\pi\epsilon_0} = C$$

$$\therefore \vec{F} = \frac{C}{r^2}\hat{r}$$

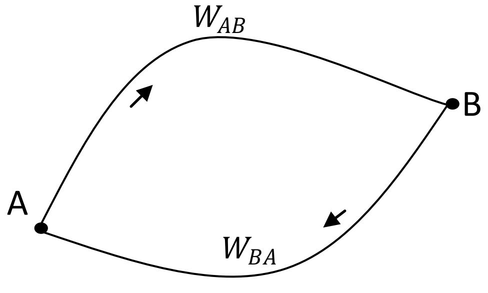
$$f(r) = \frac{C}{r^2}$$

$$\therefore f(r) \propto \frac{1}{r^2}$$

To prove that Central force is a conservative force

If the work done by a force in moving a particle from one point to another is independent of the path followed then such force is known as a conservative force. (Or)

If the work done by a force in moving a particle around a closed path is zero then the force is known as a central force.



Work done by the central force $\vec{F}$ in moving the particle from A to B is given by

$$W_{AB} = \int_A^B \vec{F}.\overrightarrow{dr}$$

$$\text{Central force} \vec{F} = f(r)\hat{r}$$

$$W_{AB} = \int_A^B f(r)\hat{r}.\overrightarrow{dr}$$

$$\vec{r}.\vec{r} = r^2$$

Differentiating on both sides $\vec{r}.\overrightarrow{dr}+\overrightarrow{dr}.\vec{r}=2r\ dr$

$$2\vec{r}.\overrightarrow{dr}=2r\ dr$$

$$\vec{r}.\overrightarrow{dr}=r\ dr$$

$$\frac{\vec{r}.\overrightarrow{dr}}{r}=dr$$

$$\boldsymbol{\hat{r}.\overrightarrow{dr}=dr}$$

$$\therefore W_{AB}=\int_A^B f(r)\hat{r}\,.\overrightarrow{dr}=\int_A^B f(r)\ dr$$

$$W_{AB}=\int_A^B f(r)\ dr$$

Value of this integral depends only on the nature of the function and the limits.

Hence $W_{BA}=\int_B^A f(r)\ dr=-\int_A^B f(r)\ dr=-W_{AB}$

$$\therefore W_{AB}+W_{BA}=0$$

Hence the work done by central force in moving a particle around the closed path is zero.

Properties of Central forces

- ✓ A force which always acts towards or away from a fixed point and whose magnitude depends only on the distance of the particle from the fixed point is known as a central force.

Central force $\boldsymbol{\vec{F}=f(r)\hat{r}}$

- ✓ Central force is a conservative force. Work done by a central force in moving a particle from one point to another is independent of the path followed.
- ✓ Under the action of a central force the torque acting on a particle is zero.
- ✓ Under the action of a central force the angular momentum of a particle remains constant.
- ✓ Under the action of a central force the aerial velocity remains constant.

Areal Velocity= $\frac{dA}{dt}=\frac{h}{2}$ =Constant

Equation of motion of a particle under the action of a central force

When Central force act on a particle, the acceleration is always in the direction of radius vector. This acceleration is known as radial acceleration.

Radial acceleration $a_r=\boldsymbol{\frac{d^2r}{dt^2}-r\left(\frac{d\theta}{dt}\right)^2}$

Under Central force, the transverse acceleration is always zero.

Transverse acceleration $a_t=\frac{1}{r}\frac{d}{dt}\left(r^2\boldsymbol{\frac{d\theta}{dt}}\right)=0$

$$r^2\frac{d\theta}{dt}=h=Constant$$

$$\frac{d\theta}{dt}=\frac{h}{r^2}$$

$$Let\quad r=\frac{1}{u}$$

$$\frac{dr}{dt}=\frac{d}{dt}\left(\frac{1}{u}\right)=-\frac{1}{u^2}\frac{du}{dt}=-\frac{1}{u^2}\frac{du}{d\theta}\frac{d\theta}{dt}=-\frac{1}{u^2}\frac{du}{d\theta}\frac{h}{r^2}=-h\frac{du}{d\theta}$$

$$\boldsymbol{\frac{dr}{dt}=-h\frac{du}{d\theta}}$$

$$\frac{d^2r}{dt^2}=\frac{d}{dt}\left(\frac{dr}{dt}\right)=\frac{d}{dt}\left(-h\frac{du}{d\theta}\right)=-h\frac{d}{dt}\left(\frac{du}{d\theta}\right)=-h\frac{d}{d\theta}\left(\frac{du}{d\theta}\right)\frac{d\theta}{dt}=-h\frac{d^2u}{d\theta^2}\frac{h}{r^2}=-h^2u^2\frac{d^2u}{d\theta^2}$$

$$\boldsymbol{\frac{d^2r}{dt^2}=-h^2u^2\frac{d^2u}{d\theta^2}}$$

$$a_r=\frac{d^2r}{dt^2}-r\left(\frac{d\theta}{dt}\right)^2=-h^2u^2\frac{d^2u}{d\theta^2}-r\ \frac{h^2}{r^4}=-h^2u^2\frac{d^2u}{d\theta^2}-h^2u^3$$

From Newton's Second law

$$F = -ma_r = -m\left(-h^2u^2\frac{d^2u}{d\theta^2} - h^2u^3\right)$$

$$F = m\left(h^2u^2\frac{d^2u}{d\theta^2} + h^2u^3\right)$$

$$\frac{F}{m} = h^2u^2\left(\frac{d^2u}{d\theta^2} + u\right)$$

$$p = h^2u^2\left(\frac{d^2u}{d\theta^2} + u\right)$$

$$\boldsymbol{\frac{d^2u}{d\theta^2} + u = \frac{p}{h^2u^2}}$$

Kepler's first law of planetary motion

Every planet revolves around the sun in an elliptical orbit with the sun at one of its foci. This is known as Kepler's first law of planetary motion.

Let a planet of mass m revolves around sun of mass M in an elliptical orbit.

Gravitational force $F = \frac{GMm}{r^2} = \frac{\mu m}{r^2}$

$$\because GM = \mu = Constant$$

$$p = \frac{F}{m} = \frac{\mu}{r^2}$$

Equation of motion of a particle under the action of Central force

$$\frac{d^2u}{d\theta^2} + u = \frac{p}{h^2u^2} = \frac{\mu}{r^2h^2u^2} = \frac{\mu}{h^2}$$

$$\frac{d^2u}{d\theta^2} + u = \frac{\mu}{h^2}$$

$$\frac{d^2u}{d\theta^2} + \left(u - \frac{\mu}{h^2}\right) = 0$$

$$\frac{d^2}{d\theta^2}\left(u - \frac{\mu}{h^2}\right) + \left(u - \frac{\mu}{h^2}\right) = 0$$

$$let u - \frac{\mu}{h^2} = X$$

$$\frac{d^2X}{d\theta^2} + X = 0$$

Solution of this differential equation

$$X = A\cos(\theta - \theta_0)$$

$$u - \frac{\mu}{h^2} = X = A\cos(\theta - \theta_0)$$

$$\boldsymbol{u = \frac{\mu}{h^2} + A\cos(\theta - \theta_0)}$$

$$u = \frac{\mu}{h^2}\left[1 + A\frac{h^2}{\mu}\cos(\theta - \theta_0)\right]$$

$$\frac{1}{r} = \frac{1 + A\frac{h^2}{\mu}\cos(\theta - \theta_0)}{\frac{h^2}{\mu}}$$

This equation is similar to the equation of a Conic.

$$\frac{1}{r} = \frac{1 + e\cos\theta}{l}$$

Eccentricity $\boldsymbol{e = \frac{Ah^2}{\mu}}$

Kinetic Energy $K.E = \frac{1}{2}m\left[\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2\right]$

$$\frac{dr}{dt} = -h\frac{du}{d\theta} and \frac{d\theta}{dt} = \frac{h}{r^2}$$

$$K.E = \frac{1}{2}m\left[h^2\left(\frac{du}{d\theta}\right)^2 + r^2\frac{h^2}{r^4}\right] = \frac{1}{2}m\left[h^2\left(\frac{du}{d\theta}\right)^2 + h^2u^2\right]$$

$$K.E = \frac{1}{2}mh^2\left[\left(\frac{du}{d\theta}\right)^2 + u^2\right]$$

$$u = \frac{\mu}{h^2} + A\cos(\theta - \theta_0)$$

$$\frac{du}{d\theta} = -A\sin(\theta - \theta_0)$$

$$K.E = \frac{1}{2}mh^2\left[A^2\sin^2(\theta - \theta_0) + \frac{\mu^2}{h^4} + A^2\cos^2(\theta - \theta_0) + \frac{2\mu A}{h^2}\cos(\theta - \theta_0)\right]$$

$$\boldsymbol{K.E = \frac{1}{2}mh^2\left[A^2 + \frac{\mu^2}{h^4} + \frac{2\mu A}{h^2}\cos(\theta - \theta_0)\right]}$$

Potential Energy $P.E = \int_\infty^r F dr = \int_\infty^r \frac{\mu m}{r^2}dr = \mu m\int_\infty^r \frac{1}{r^2}dr = \mu m\left(-\frac{1}{r}\right)_\infty^r = -\mu m\left(\frac{1}{r}\right)_\infty^r$

$$P.E = -\mu m\frac{1}{r} = -\mu m u$$

$$P.E = -\mu m\left[\frac{\mu}{h^2} + A\cos(\theta - \theta_0)\right]$$

$$= -m\left(\mu\frac{\mu}{h^2} + \mu A\cos(\theta - \theta_0)\right) = -m\left(\frac{\mu^2}{h^2} + \mu A\cos(\theta - \theta_0)\right)$$

$$= -\frac{1}{2}mh^2\left(\frac{2}{h^2}\frac{\mu^2}{h^2} + \frac{2}{h^2}\mu A\cos(\theta - \theta_0)\right) = -\frac{1}{2}mh^2\left(\frac{2\mu^2}{h^4} + \frac{2\mu A}{h^2}\cos(\theta - \theta_0)\right)$$

$$\boldsymbol{P.E = -\frac{1}{2}mh^2\left[\frac{2\mu^2}{h^4} + \frac{2\mu A}{h^2}\cos(\theta - \theta_0)\right]}$$

$$E = K.E + P.E = \frac{1}{2}mh^2\left[A^2 + \frac{\mu^2}{h^4} + \frac{2\mu A}{h^2}\cos(\theta - \theta_0)\right] - \frac{1}{2}mh^2\left(\frac{2\mu^2}{h^4} + \frac{2\mu A}{h^2}\cos(\theta - \theta_0)\right)$$

$$E = \frac{1}{2}mh^2\left[A^2 - \frac{\mu^2}{h^4}\right]$$

$$\frac{2E}{mh^2} = A^2 - \frac{\mu^2}{h^4}$$

$$A^2 = \frac{\mu^2}{h^4} + \frac{2E}{mh^2} = \frac{\mu^2}{h^4}\left(1 + \frac{2E}{mh^2}\frac{h^4}{\mu^2}\right) = \frac{\mu^2}{h^4}\left(1 + \frac{2Eh^2}{m\mu^2}\right)$$

$$A = \frac{\mu}{h^2}\sqrt{1 + \frac{2Eh^2}{m\mu^2}}$$

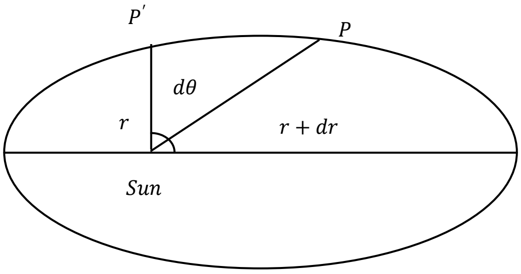
$$\boldsymbol{e = \frac{Ah^2}{\mu} = \sqrt{1 + \frac{2Eh^2}{m\mu^2}}}$$

For a bound System $E < 0$. Hence Eccentricity $e < 1$.
Hence the orbit is an ellipse.

Kepler's Second law of planetary motion:

The area velocity of a planet always remains constant. This is known as Kepler's Second law of planetary motion.

Consider a planet is moved from P to P' in a time Δt as shown in figure.



$$\text{Area } dA = \text{Area of the triangle} = \frac{1}{2}\, r\,(r + dr)\, \sin d\theta$$

$$If\, \Delta t \to 0, then\;\; r\,(r + dr) \approx r^2 and\, \sin d\theta = d\theta$$

$$dA = \frac{1}{2} r^2 d\theta$$

$$\text{Areal Velocity} = \frac{dA}{dt} = \frac{\frac{1}{2} r^2 d\theta}{dt} = \frac{1}{2}\left(r^2 \frac{\boldsymbol{d\theta}}{\boldsymbol{dt}}\right)$$

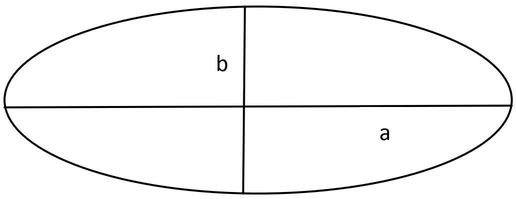
$$r^2 \frac{\boldsymbol{d\theta}}{\boldsymbol{dt}} = \frac{h}{2} = \text{Constant}$$

$$\text{Areal Velocity} = \frac{dA}{dt} = \frac{h}{2} = \text{Constant}$$

Kepler's third law of planetary motion:

Square of the time period of a planet is directly proportional to the cube of the length of its semi-major axis.

$$T^2 \propto a^3$$



$$\text{Time period } T = \frac{\text{Area swept in one revolution}}{\text{Areal velocity}} = \frac{\pi ab}{\frac{h}{2}} = \frac{2\pi ab}{h}$$

$$\text{Length of semi latus rectum}\quad l = \frac{h^2}{\mu} = \frac{b^2}{a}$$

$$\frac{h^2}{\mu} = \frac{b^2}{a}$$

$$h^2 = b^2 \frac{\mu}{a}$$

$$h = b\sqrt{\frac{\mu}{a}}$$

$$\therefore\; T = \frac{2\pi ab}{b\sqrt{\frac{\mu}{a}}} = \frac{2\pi a\sqrt{a}}{\sqrt{\mu}}$$

$$T^2 = \frac{4\pi^2 a^3}{\mu} = \left(\frac{4\pi^2}{\mu}\right) a^3$$

$$T^2 \propto a^3$$

G.P.S

- G.P.S. means Global Positioning System
- G.P.S was developed by the American government in the year 1973.
- G.P.S is used to know the position and time coordinates of any point on the Earth's surface or near it.
- G.P.S is based on the network of 31 satellites orbiting Earth in different orbits. The position and time coordinates are obtained basing on the information received from at least 4 satellites in the network of 31 satellites.
- Accuracy of G.P.S ranges from 10m to 100m.
- There are mainly three parts in G.P.S System.
    1. Space segment
    2. Control segment
    3. User segment
- G.P.S is widely used in different fields like Aviation, Marine, Defence, Transportation, Industry, Agriculture etc.

Unit-III

3. Relativistic Mechanics

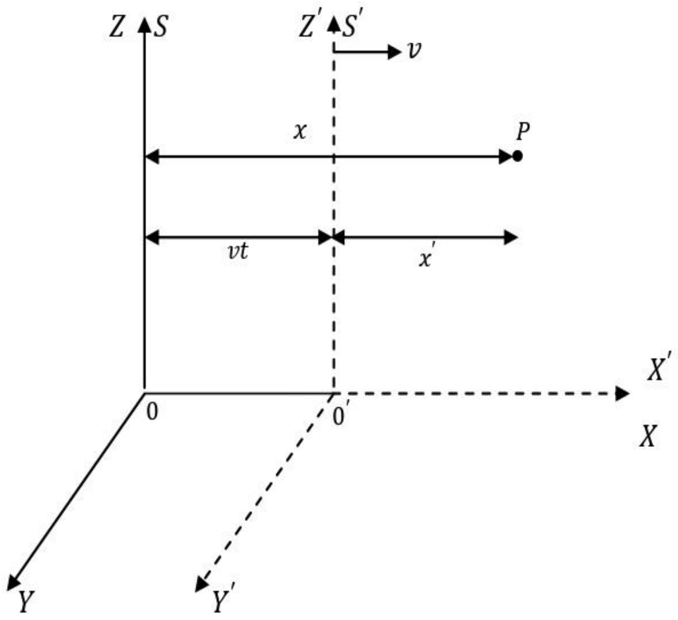
Postulates of Special theory of relativity:

- ✓ *Laws of Physics remain the same for all observers in uniform motion relative to one another*
- ✓ *Speed of light is the same for all observers in uniform motion relative to one another.*

Inertial Frame of Reference: *A frame of reference in which Newton's laws of motion are valid is known as an inertial frame of reference.*

Non-Inertial Frame of Reference: *A frame of reference in which Newton's Laws are not valid is known as a Non-inertial frame of reference.*

Galilean Transformation



Consider two inertial frames of reference S, S'. Frame S' is moving with a velocity v along the positive X-axis relative to the frame S. Let the two frames of reference S, S' coincide at time $t = 0$.

Let be the co-ordinates of the point P with respect to the frames S, S' are (x, y, z, t) and (x', y', z', t').

From Figure,

$$x = x' + vt$$
$$x' = x - vt$$
Similarly
$$y' = y,$$
$$z' = z,$$
$$t' = t$$

The above equations are called Galilean Transformation equations.

Inverse Galilean Transformation equations are

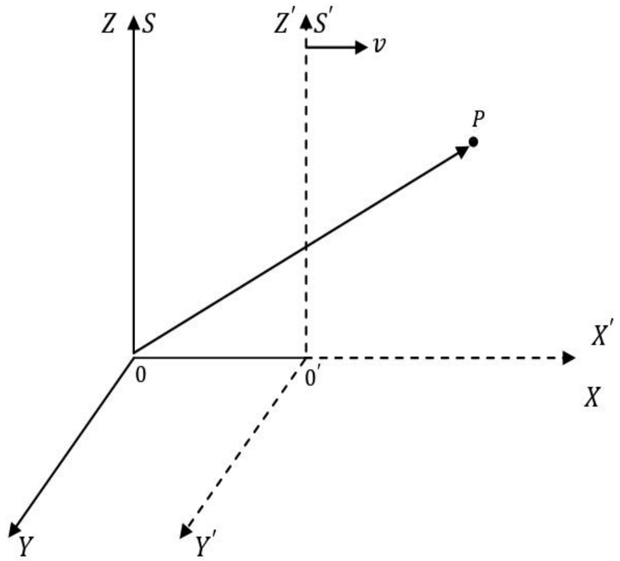
$$x = x' + vt'$$
$$y = y'$$
$$z = z'$$
$$t = t'$$

- ➢ *Space interval is invariant under Galilean transformation*
- ➢ *Time interval is invariant under Galilean Transformation.*
- ➢ *Laws of mechanics are invariant under Galilean Transformation.*

Lorentz Transformation

Consider two inertial frames of reference S, S'. Frame S' is moving with a velocity v along the positive X-axis relative to the frame S. Let the two frames of reference S, S' coincide at time $t = 0$. Let be the co-ordinates of the point P with respect to the frames S, S' are (x, y, z, t) and (x', y', z', t').

Let a beam of light is emitted from the origin O at time $t = o$. The beam of light reaches the point P after a time.



Relative to frame S, $C = \frac{\text{Distance}}{\text{Time}} = \frac{\sqrt{(x^2+y^2+z^2)}}{t}$

Relative to Frame S', $C = \frac{\text{Distance}}{\text{Time}} = \frac{\sqrt{x'^2+y'^2+z'^2}}{t}$

$$C = \frac{\sqrt{(x^2+y^2+z^2)}}{t}$$

$$c^2t^2 = x^2 + y^2 + z^2$$

$$x^2 + y^2 + z^2 - c^2t^2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots 1$$

$$C = \frac{\sqrt{x'^2 + y'^2 + z'^2}}{t'}$$

$$c^2t'^2 = x'^2 + y'^2 + z'^2$$

$$x'^2 + y'^2 + z'^2 - c^2t'^2 = 0 \ldots\ldots\ldots\ldots\ldots 2$$

From equations 1 and 2

$$x^2 + y^2 + z^2 - c^2t^2 = x'^2 + y'^2 + z'^2 - c^2t'^2$$

$$y' = y,$$

$$z' = z$$

$$x^2 - c^2t^2 = x'^2 - c^2t'^2 \ldots\ldots\ldots\ldots\ldots 3$$

From Galilean Transformation

$$x' = x - vt$$

$$Let \quad x' = k(x - vt) \ldots\ldots\ldots\ldots 4$$

Inverse Galilean Transformation

$$x = k(x' + vt')$$

$$x = k[k(x - vt) + vt']$$

$$\frac{x}{k} = k(x - vt) + vt'$$

$$vt' = \frac{x}{k} - k(x - vt)$$

$$vt' = \frac{x}{k} - kx + kvt$$

$$t' = \frac{x}{kv} - \frac{kx}{v} + kt$$

$$t' = kt + \left(\frac{x}{kv} - \frac{kx}{v}\right)$$
$$t' = kt - \frac{x}{v}\left(k - \frac{1}{k}\right)$$
$$t' = kt - \frac{kx}{v}\left(1 - \frac{1}{k^2}\right)$$
$$t' = k\left[t - \frac{x}{v}\left(1 - \frac{1}{k^2}\right)\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 5$$

From equation 3, $x^2 - c^2t^2 = x'^2 - c^2t'^2$

$$x^2 - c^2t^2 = k^2(x - vt)^2 - c^2k^2\left[t - \frac{x}{v}\left(1 - \frac{1}{k^2}\right)\right]^2$$

Comparing the coefficients of t^2 *on both sides,*

$$-c^2 = k^2v^2 - c^2k^2$$
$$c^2 = c^2k^2 - k^2v^2$$
$$c^2 = k^2(c^2 - v^2)$$
$$k^2 = \frac{c^2}{c^2 - v^2} = \frac{1}{1 - v^2/c^2}$$
$$k = \frac{1}{\sqrt{1 - v^2/c^2}}$$

From equation 4,

$$x' = k(x - vt)$$
$$x' = \frac{(x - vt)}{\sqrt{1 - v^2/c^2}}$$

From equation 5,

$$t' = k\left[t - \frac{x}{v}\left(1 - \frac{1}{k^2}\right)\right] = \frac{1}{\sqrt{1 - v^2/c^2}}\left[t - \frac{x}{v}\left(1 - \frac{c^2 - v^2}{c^2}\right)\right]$$
$$= \frac{1}{\sqrt{1 - v^2/c^2}}\left[t - \frac{x}{v}\left(\frac{c^2}{v^2}\right)\right] = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$
$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

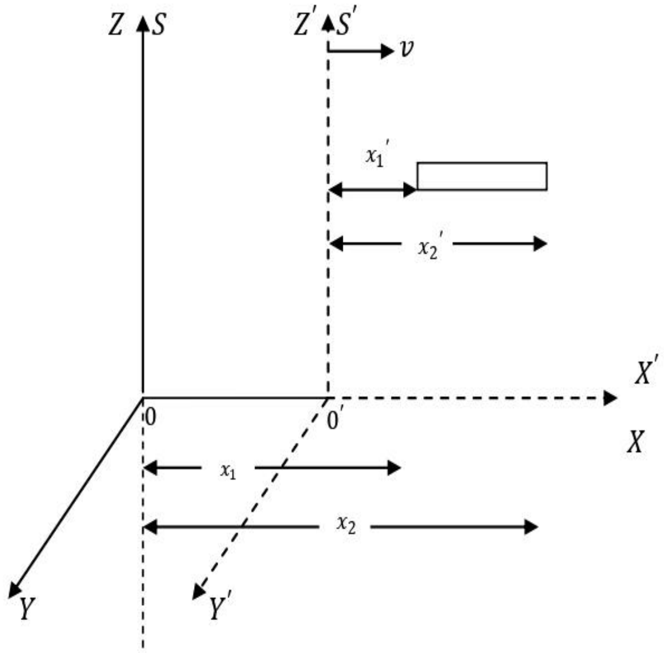
Lorentz Transformation Equations

$$x' = \frac{(x - vt)}{\sqrt{1 - v^2/c^2}}$$
$$y' = y,$$
$$z' = z,$$
$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

Length Contraction or Lorentz-Fitzgerald Contraction

Consider two inertial frames of reference S, S'. *Frame* S' *is moving with a velocity* v *along the positive* X*-axis relative to the frame* S. *Let the two frames of reference* S, S' *coincide at time* $t = 0$.

Let a rod of length l *is placed in the reference frame* S' *with its length parallel to* X*-axis. Co-ordinates of the ends of the rod with respect to the frames* S, S' *are* (x_1, x_2) *and* (x_1', x_2').



Length of the rod in frame S

$$l = x_2 - x_1$$

Length of the rod in frame S'

$$l' = x_2' - x_1'$$

From Lorentz transformation

$$x' = \frac{(x - vt)}{\sqrt{1 - v^2/c^2}}$$

$$l' = x_2' - x_1' = \frac{(x_2 - vt)}{\sqrt{1 - v^2/c^2}} - \frac{(x_1 - vt)}{\sqrt{1 - v^2/c^2}} = \frac{x_2 - x_1}{\sqrt{1 - v^2/c^2}}$$

$$l' = \frac{l}{\sqrt{1 - v^2/c^2}}$$

$$l = l'\sqrt{1 - v^2/c^2}$$

Hence a moving rod appears to be contracted for a stationary observer.

Case (i): $When\ v \ll c \quad \frac{v^2}{c^2} \sim 0$

$$\therefore l = l'\sqrt{1} = l'$$
$$l = l'$$

Case(ii): $When\ v\ is$ *comparable to* $c, l = l'\sqrt{1 - v^2/c^2}$

$$\therefore l < l'$$

Moving rod appears to be contracted for a stationary observer

Case(iii): $When\ v = c, \quad \frac{v^2}{c^2} = 1$

$$l = l'(0) = 0$$

$$\therefore l = 0$$

$When\ v > c, \quad l = Complex\ Number$

- ✓ *Hence no object can travel faster than the speed of light.*

Time Dilation

Consider two inertial frames of reference S, S' . Frame S' is moving with a velocity v along the positive X-axis relative to the frame S. Let the two frames of reference S, S' coincide at time $t = 0$.

Let a clock be placed in the frame$S$.

Time interval in frame S is $\Delta t = t_2 - t_1$

Time interval in frame S' is $\Delta t' = t_2' - t_1^1$

From Lorentz Transformation

$$t' = \frac{t - {vx}/{c^2}}{\sqrt{1 - {v^2}/{c^2}}}$$

$$\Delta t' = t_2' - t_1^1 = \frac{t_2 - {vx}/{c^2}}{\sqrt{1 - {v^2}/{c^2}}} = \frac{t_1 - {vx}/{c^2}}{\sqrt{1 - {v^2}/{c^2}}} = \frac{t_2 - t_1}{\sqrt{1 - {v^2}/{c^2}}} = \frac{\Delta t}{\sqrt{1 - {v^2}/{c^2}}}$$

$$\therefore \Delta t' = \frac{\Delta t}{1 - {v^2}/{c^2}} = k\Delta t$$

Case (i): When $v \ll c$ $\quad \frac{v^2}{c^2} \sim 0$

$\therefore \Delta t' = \Delta t$

Case(ii): When $v$is comparable to c

$$\Delta t' > \Delta t$$

Hence the time interval of a moving observer is more than the time interval of a stationery observer.

Case(iii): When $v = c$, $\quad \frac{v^2}{c^2} = 1$

$$\Delta t' = \infty$$

When $v > c$, $\quad \Delta t' =$*Complex Number*

- ✓ *Hence no object can travel faster than the speed of light.*

Einstein's Mass-Energy Equivalence

From Newton's Second law

$$F = \frac{dP}{dt} = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt}$$

By Work-Energy theorem, work done is equal to the change in kinetic energy.

$$W = F.dx = dK$$

$$dK = F.dx = \left(m\frac{dv}{dt} + v\frac{dm}{dt}\right)dx$$

$$= m\frac{dv}{dt}dx + v\frac{dm}{dt}dx$$

$$= m\,dv\,\frac{dx}{dt} + v\,dm\,\frac{dx}{dt}$$

$$= m\,v\,dv + v^2dm$$

$$\therefore dK = m\,v\,dv + v^2dm$$

Relativistic mass

$$m = \frac{m_0}{\sqrt{1 - {v^2}/{c^2}}}$$

$$m^2 = \frac{m_0^2}{1 - v^2/c^2} = \frac{m_0^2}{\frac{c^2 - v^2}{c^2}} = \frac{m_0^2 c^2}{c^2 - v^2}$$

$$m^2(c^2 - v^2) = m_0^2 c^2$$
$$m^2 c^2 - m^2 v^2 = m_0^2 c^2$$
$$2mc^2 dm - (m^2\, 2v\, dv + v^2\, 2m\, dm) = 0$$
$$2mc^2 dm = (m^2\, 2v\, dv + v^2\, 2m\, dm)$$
$$c^2 dm = m\, v\, dv + v^2\, dm$$
$$dK = c^2 dm$$
$$\int dK = c^2 \int_{m_0}^{m} dm$$
$$K = c^2 (m)_{m_0}^{m}$$
$$K = c^2(m - m_0)$$

The above equation gives the relativistic kinetic energy of a moving body.

Energy at rest is given by

$$m_0 c^2$$

Total energy

$$E = c^2(m - m_0) + m_0 c^2 = mc^2$$

The above equation gives Einstein's mass-energy equivalence.

Hence Mass and Energy are not two different physical quantities. Mass can be converted in to energy and vice-versa.

Addition of Velocities or Transformation of Velocities

Consider two inertial frames of reference S, S'. Frame S' is moving with a velocity v along the positive X-axis relative to the frame S. Let the two frames of reference S, S' coincide at time $t = 0$.

In reference frame S, an object moves a distance dx in time dt. Similarly in reference frame S', the object moves a distance dx' in time dt'.

Velocity in Reference frame S $\quad u = \frac{dx}{dt}$

Velocity in Reference frame S' $\quad u' = \frac{dx'}{dt'}$

From Lorentz Transformation

$$x' = k(x - vt)$$
$$t' = k\left(t - {vx}/{c^2}\right)$$

From Inverse Lorentz Transformation

$$x = k(x' + vt'),\ t = k\left(t' + {vx'}/{c^2}\right)$$
$$dx = k(dx' + v\, dt'),\ dt = k\left(dt' + {v dx'}/{c^2}\right)$$
$$u = \frac{dx}{dt} = \frac{k(dx' + v\, dt')}{k\left(dt' + {vdx'}/{c^2}\right)} = \frac{(dx' + v\, dt')}{\left(dt' + {vdx'}/{c^2}\right)}$$
$$= \frac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2}\frac{dx'}{dt'}}$$
$$u = \frac{u' + v}{1 + {u'v}/{c^2}}$$

The above equation represents the relativistic law of addition of velocities.

Case(i) : When $u' \ll c,\ v \ll c$

$$\frac{u'v}{c^2} \sim 0$$

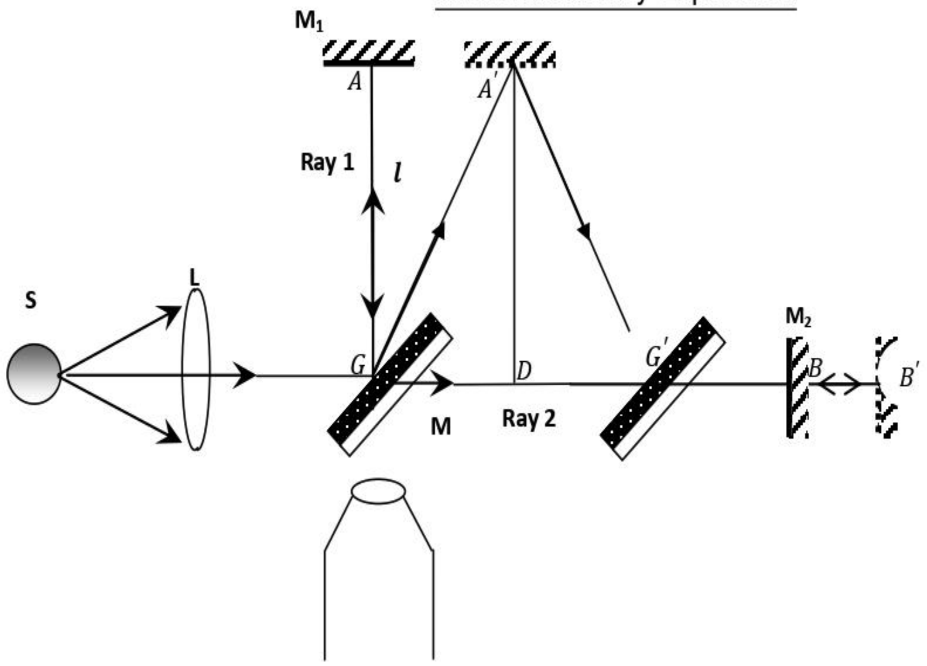
$$\therefore u = u' + v$$

Case(ii): When $u' = c, v = c$

$$u = \frac{c+c}{1 + {c.c}/{c^2}} = \frac{2c}{2} = c$$
$$u = c$$

Hence addition of velocity of to the velocity of light reproduces the velocity of light.

Michelson-Morley Experiment



Aim: Aim of Michelson-Morley experiment is to determine the velocity of Earth relative to Ether.

Michelson-Morley Interferometer is shown in figure. Light emitted from the monochromatic source S *falls on the half silvered glass plate* G. *The glass plate* G *is oriented at an angle of* 45^0 *to the incident light. Hence the light incident on the glass plate* G *is divided in to two perpendicular beams of light. The two beams of light are reflected back from the two mirrors* M_1, M_2 *and meet at* G *to produce interference pattern. The interference pattern can be observed through the telescope*T.

Since the apparatus is moving with a velocity v *along with the Earth, the optical paths of two beams are not equal. The two beams are reflected at the points* A', B' *instead of* A, B *and interfere at* G'.

From $\Delta\, GA'D$ $(GA')^2 = (A'D)^2 + (GD)^2$

$$c^2t^2 = l^2 + v^2t^2$$
$$l^2 = (c^2 - v^2)t^2$$
$$t^2 = \frac{l^2}{(c^2 - v^2)}$$
$$t = \frac{l}{\sqrt{c^2 - v^2}} = \frac{l}{c\sqrt{1 - {v^2}/{c^2}}}$$
$$= \frac{l}{c}\left(1 - {v^2}/{c^2}\right)^{-1/2} = \frac{l}{c}\left(1 + {v^2}/{2c^2}\right)$$

Hence the time taken by the light beam 1 to reach G'

$$t_1 = 2t = \frac{2l}{c}\left(1 + {v^2}/{2c^2}\right)$$

Let be t_2 *the time taken by the light beam 2 to reach the glass plate* G'.

Velocity of light beam from G *to* B' *is* $(c - v)$ *and from* B' *to* G' *is* $(c + v)$.

$$t_2 = \frac{l}{c-v} + \frac{l}{c+v} = l\left(\frac{1}{c-v} + \frac{1}{c+v}\right) = l\left(\frac{2c}{c^2 - v^2}\right)$$

$$\frac{l(2c)}{c^2\left(1 - {v^2}/{c^2}\right)} = \frac{2l}{c}\left(1 - {v^2}/{c^2}\right)^{-1} = \frac{2l}{c}\left(1 + {v^2}/{c^2}\right)$$

$$t_2 = \frac{2l}{c}\left(1 + {v^2}/{c^2}\right)$$

Time lag between the two beams

$$\Delta t = t_2 - t_1$$

$$= \frac{2l}{c}\left(1 + {v^2}/{c^2}\right) - \frac{2l}{c}\left(1 + {v^2}/{2c^2}\right)$$

$$= \frac{2l}{c}\left(1 + {v^2}/{c^2} - 1 - {v^2}/{2c^2}\right)$$

$$= \frac{2l}{c}\left(\frac{v^2}{2c^2}\right)$$

$$= \frac{lv^2}{c^3}$$

$$\Delta t = \frac{lv^2}{c^3}$$

Path difference $= c.\Delta t = c.\frac{lv^2}{c^3} = \frac{lv^2}{c^2}$

Path difference in terms of Wavelength $= \frac{lv^2}{\lambda c^2}$

Mirrors M_1, M_2 *are interchanged by rotating the apparatus by* 90^0

Path difference $= -\frac{lv^2}{\lambda c^2}$

Resultant Path difference $= \frac{lv^2}{\lambda c^2} - \left(-\frac{lv^2}{\lambda c^2}\right) = \frac{2lv^2}{\lambda c^2}$

Hence Fringe Shift $\Delta n = \frac{2lv^2}{\lambda c^2}$

In Michelson-Morley Experiment, $l = 10m, v = 3 \times 10^4\ m/s\ , \lambda = 5000\ \times 10^{-10} m, c = 3\ \times 10^8\ m/s$

$$\therefore \Delta n = \frac{2 \times 10 \times (3 \times 10^4)^2}{5000 \times 10^{-10} \times (3 \times 10^8)^2} = 0.4$$

Hence a fringe shift of 0.4 *was expected. But Michelson-Morley observed a fringe shift of only 0.001. This is known as Null Result.*

Significance of Null Result:

- *It is impossible to measure the speed of Earth relative to Ether. Hence the concept of Ether is rejected.*
- *Speed of light in vacuum is the same for all observers.*

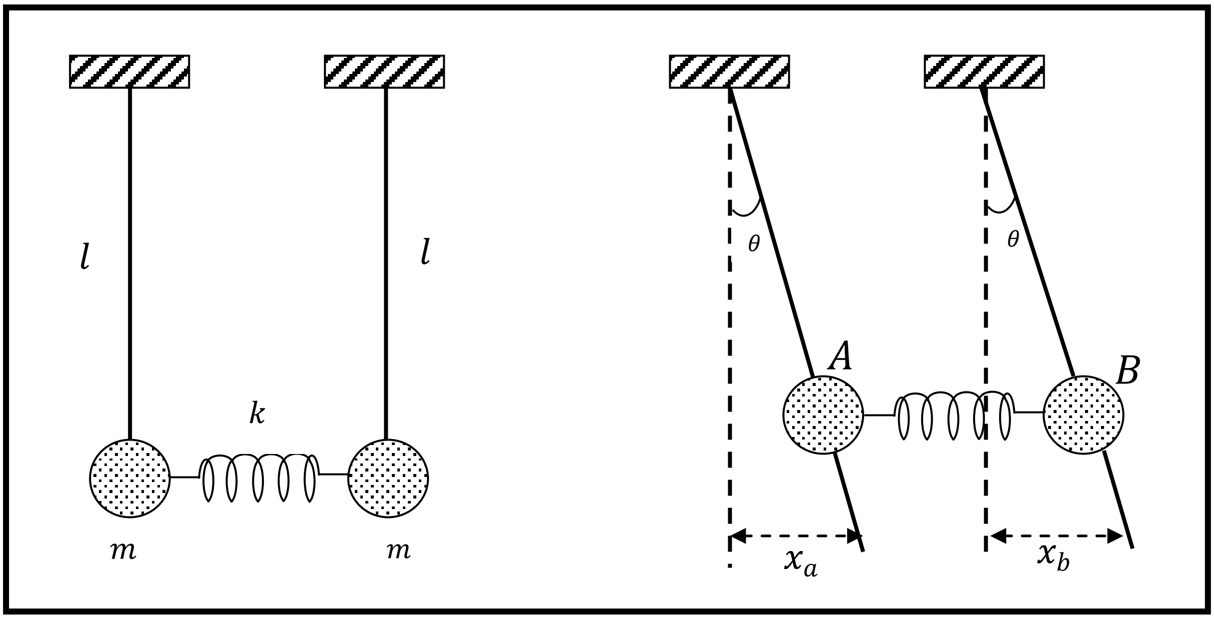
Unit-IV
6. Coupled Oscillations

Two Coupled Oscillators-Normal Coordinates & Normal Modes

The oscillations of a system of two pendulums coupled by a spring are known as Coupled Oscillations.

Consider two Simple Pendulums ‘A’ & ‘B’ each of mass ‘m’ and length ‘l’ coupled by a spring of force constant ‘k’ as shown in figure. Let us assume that the Pendulum ‘B’ is drawn aside while the Pendulum ‘A’ is fixed and then both are released. The amplitude of Pendulum ‘B’ decreases gradually and the amplitude of Pendulum ‘A’ increases. After some time, the amplitudes of ‘A’ and ‘B’ are equal. The amplitude of ‘B’ continues to decrease till it becomes zero while the amplitude of ‘A’ becomes maximum. In this way the energy of Pendulum is completely transferred to the Pendulum ‘A’.

Let x_a and x_b be the displacements of Pendulums ‘A’ and ‘B’ respectively.



Two forces act on the Pendulum ‘B’.

1. Restoring force due to gravity

$$F_1 = -mg\, sin\theta = -mg\left(\frac{x_b}{l}\right) \longrightarrow (1)$$

2. Return force due to stretching of spring

$$F_2 = -k(x_b - x_a) \longrightarrow (1)$$

Hence the equations of motions of Pendulums ‘A’ and ‘B’ are given by

$$m\frac{d^2x_a}{dt^2} = -\frac{mg}{l}x_a + k(x_b - x_a)$$

$$m\frac{d^2x_b}{dt^2} = -\frac{mg}{l}x_b - k(x_b - x_a)$$

Dividing the above two equations with ‘m’

$$\frac{d^2x_a}{dt^2} = -\frac{g}{l}x_a + \frac{k}{m}(x_b - x_a)$$

$$\frac{d^2x_b}{dt^2} = -\frac{g}{l}x_b - \frac{k}{m}(x_b - x_a)$$

If ω_0 is the angular frequency of coupled oscillations, then

$$\omega_0 = \sqrt{\frac{g}{l}}$$

$$\frac{d^2x_a}{dt^2} = -\omega_0^2\, x_a + \frac{k}{m}(x_b - x_a) \longrightarrow (3)$$

$$\frac{d^2x_b}{dt^2} = -\omega_0^2\, x_b - \frac{k}{m}(x_b - x_a) \longrightarrow (4)$$

Adding the equations 3 & 4,

$$\frac{d^2}{dt^2}(x_a + x_b) = -\omega_0^2(x_a + x_b)$$

$$\frac{d^2}{dt^2}(x_a + x_b) + \omega_0^2\,(x_a + x_b) = 0$$

$$\text{Let } x_a + x_b = X$$

$$\frac{d^2X}{dt^2} + \omega_0^2\, X = 0$$

Subtracting the equations 3 & 4

$$\frac{d^2}{dt^2}(x_b - x_a) = -\omega_0^2(x_b - x_a) - 2\left(\frac{k}{m}\right)(x_b - x_a)$$

$$\frac{d^2}{dt^2}(x_b - x_a) + \omega_0^2(x_b - x_a) + 2\left(\frac{k}{m}\right)(x_b - x_a) = 0$$

$$\frac{d^2}{dt^2}(x_b - x_a) + (\omega_0^2 + \frac{2k}{m})(x_b - x_a) = 0$$

$$\text{Let } \quad Y = x_b - x_a$$

$$\frac{d^2\mathrm{Y}}{dt^2} + (\omega_0^2 + \frac{2k}{m})\mathrm{Y} = 0$$

$$\frac{d^2\mathrm{Y}}{dt^2} + \omega_0^2 Y = 0$$

$$\omega_2^2 = \omega_0^2 + \frac{2k}{m}$$

Case (i):

If $x_a = x_b$, the angular frequency of oscillation is given by

$$\boldsymbol{\omega_1 = \omega_0 = \sqrt{\frac{g}{l}}}$$

In this case, the frequency of the coupled system is equal to the natural frequency of the pendulums when they are separate. This is called first normal mode.

In this case, the equation of motion of the system is described by a linear differential equation containing only one dependent variable $Y = x_b + x_a$. This parameter is called the Normal Coordinate.

Case(ii):

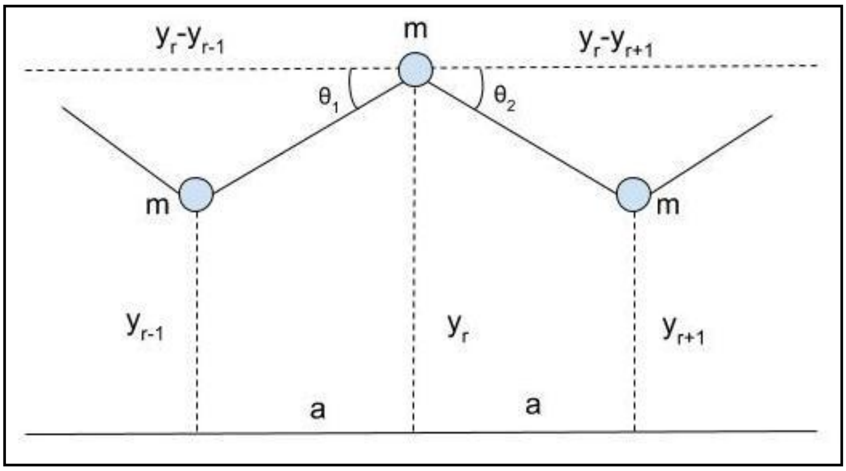
If $x_a = -x_b$, the angular frequency of oscillation is given by

$$\boldsymbol{\omega_2 = \sqrt{\omega_0^2 + \frac{2k}{m}}}$$

Hence $\omega_2 > \omega_1$. The frequency of oscillation of the coupled system is greater than the natural frequency of the pendulums when they are separate. This is called second normal mode.

In this case, the equation of motion of the system is described by a linear differential equation containing only one dependent variable $Y = x_b - x_a$. This parameter is also called the Normal Coordinate.

N-Coupled Oscillators- Theory & Wave Equation



Consider a stretched elastic spring containing 'N' identical particles each of mass 'm' separated by a distance 'a' from each other. The total length of the string is $(n+1)a$. As shown in figure, let Y_r, Y_{r-1} and Y_{r+1} be the displacements of rth, (r-1)th and (r+1)th particles.

The resultant force on the rth particle in the Y-direction is given by

$$F_r = -T(\sin\theta_1 + \sin\theta_2)$$

$$m\frac{d^2y_r}{dt^2} = -T(\sin\theta_1 + \sin\theta_2)$$

$$\sin\theta_1 = \frac{y_r - y_{r-1}}{a}$$

$$\sin\theta_2 = \frac{y_r - y_{r+1}}{a}$$

$$m\frac{d^2y_r}{dt^2} = -T\left[\frac{y_r - y_{r-1}}{a} + \frac{y_r - y_{r+1}}{a}\right]$$

$$\frac{d^2y_r}{dt^2} = \frac{-T}{ma}\left[\frac{y_r - y_{r-1}}{a} + \frac{y_r - y_{r+1}}{a}\right]$$

$$\boldsymbol{\frac{d^2y_r}{dt^2} = \frac{T}{ma}(y_{r-1} - 2y_r + y_{r+1})}$$

This is the equation of motion of N-Coupled Oscillators.

$$\text{Let } y_r = A_r e^{i\omega t}$$

Where $\boldsymbol{A_r}$ is the amplitude.

$$\text{Then } y_{r+1} = A_{r+1}\, e^{i\omega t}$$

$$y_{r-1} = A_{r-1}\, e^{i\omega t}$$

Substituting these values in equation of motion,

$$-\omega^2 A_r\, e^{i\omega t} = \frac{T}{ma}(A_{r-1} - 2A_r + A_{r+1})\, e^{i\omega t}$$

$$-\omega^2 A_r = \frac{T}{ma}(A_{r-1} - 2A_r + A_{r+1})$$

$$-\frac{ma\omega^2}{T}A_r = (A_{r-1} - 2A_r + A_{r+1})$$

$$2A_r - \frac{ma\omega^2}{T} - A_{r-1} - A_{r+1} = 0$$

$$-A_{r-1} + \left(2 - \frac{ma\omega^2}{T}\right)A_r - A_{r+1} = 0$$

Applying the boundary conditions

$$\boldsymbol{y_0 = A_0 = 0} \text{ and } \boldsymbol{y_{n+1} = A_{n+1} = 0}$$

When $\boldsymbol{r = 1}$

$$\left(2 - \frac{ma\omega^2}{T}\right)A_1 - A_2 = 0$$

When $\boldsymbol{r = 2}$

$$-A_1 + \left(2 - \frac{ma\omega^2}{T}\right)A_2 - A_3 = 0$$

When $\boldsymbol{r = n}$

$$-A_{n-1} + \left(2 - \frac{ma\omega^2}{T}\right)A_n - A_{n+1} = 0$$

$$-A_{n-1} + \left(2 - \frac{ma\omega^2}{T}\right)A_n = 0$$

Solving the above equation, we will get 'n' different values for frequency. Hence the number of normal modes are equal to the number of particles.

If $\boldsymbol{n = 1}$

$$\left(2 - \frac{ma\omega^2}{T}\right)A_1 = 0$$

$$\omega^2 = \frac{2T}{ma}$$

Hence a single oscillator has only one allowed frequency of vibration

$$\omega_1^2 = \frac{T}{ma}$$

If $\boldsymbol{n = 2}$, string length is $\boldsymbol{3a}$.

$$\left(2 - \frac{ma\omega^2}{T}\right)A_1 - A_2 = 0$$

$$-A_1 + \left(2 - \frac{ma\omega^2}{T}\right)A_2 = 0$$

We have two normal mode of frequencies given by

$$\omega_1^2 = \frac{T}{ma}$$

$$\omega_2^2 = \frac{3T}{ma}$$

Wave Equation:

Equation of motion of r^th particle is given by

$$\frac{d^2y_r}{dt^2} = \frac{T}{ma}(y_{r-1} - 2y_r + y_{r+1})$$

When the separation between the particles $a = \delta x \to 0$

$$\frac{d^2y_r}{dt^2} = \frac{T}{m}\left(\frac{y_{r-1} - 2y_r + y_{r+1}}{\delta x}\right)$$

$$= \frac{T}{m}\left[\left(\frac{y_{r+1} - y_r}{\delta x}\right) - \left(\frac{y_r - y_{r-1}}{\delta x}\right)\right]$$

$$= \frac{T}{m}\left[\left(\frac{\delta y}{\delta x}\right)_{r+1} - \left(\frac{\delta y}{\delta x}\right)_{r}\right]$$

$$\text{But } \left(\frac{dy}{dx}\right)_{x+dx} - \left(\frac{dy}{dx}\right)_{x} = \frac{d^2y}{dx^2}\, dx$$

$$\frac{d^2\text{y}}{dt^2} = \frac{T}{m}\frac{d^2y}{dx^2}\, dx$$

$$\frac{d^2\text{y}}{dt^2} = \frac{T}{\rho}\frac{d^2y}{dx^2}$$

General Wave Equation is given by

$$\frac{d^2\text{y}}{dt^2} = v^2\,\frac{d^2y}{dx^2}$$

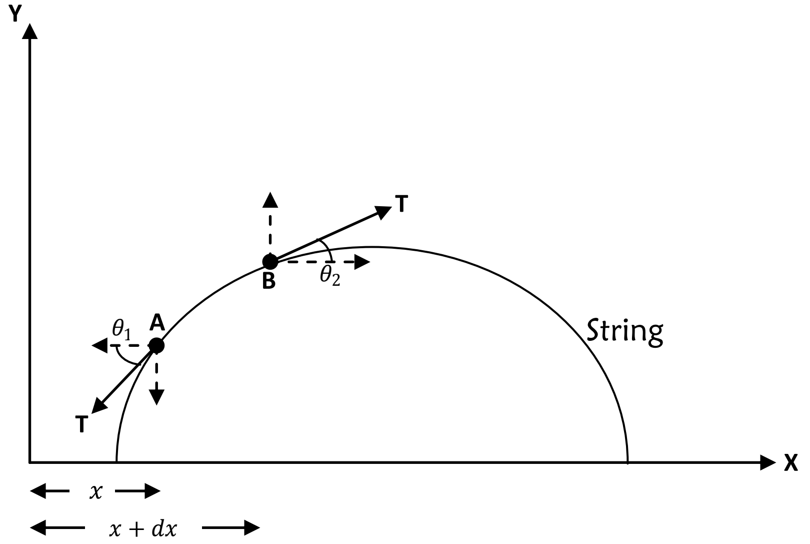
$$\text{Velocity } \boldsymbol{v} = \sqrt{\frac{\boldsymbol{T}}{\boldsymbol{\rho}}}$$

This is called the wave equation of N-Coupled Oscillators.

Unit-V

7. Vibrating Strings

Velocity of transverse wave along a stretched string (or) Transverse wave propagation along a stretched string



Consider a stretched string under a tension T along X −axis as shown in figure. If the string is plucked in a direction perpendicular to its length, transverse vibrations are produced in the string. Consider a small differential element AB of length dx between x and $x + dx$. Let y be the displacement of the string at any time t. Let θ_1, θ_2 be the angles made by the tension with the X −axis at the points A, B. The horizontal and vertical components of tension T at the point A are $T\cos\theta_1$ and $T\sin\theta_1$. Similarly the horizontal and vertical components of tension T at the point B are $T\cos\theta_2$ and $T\sin\theta_2$. The horizontal components of tension $T\cos\theta_1$ and $T\cos\theta_2$ are equal and opposite. Hence the net force along $X-$ axis is zero. The resultant vertical force in the upward direction is given by

$$F_y = T\sin\theta_2 - T\sin\theta_1 = T(\sin\theta_2 - \sin\theta_1)$$

If the displacement of AB is small, then the values of θ_1, θ_2 are small.

$$\sin\theta_1 = \tan\theta_1 = \left(\frac{\partial y}{\partial x}\right)_x$$

$$\sin\theta_2 = \tan\theta_2 = \left(\frac{\partial y}{\partial x}\right)_{x+dx}$$

$$F_y = T\left[\left(\frac{\partial y}{\partial x}\right)_{x+dx} - \left(\frac{\partial y}{\partial x}\right)_x\right] \qquad (1)$$

According to Taylor's series

$$\left(\frac{\partial y}{\partial x}\right)_{x+dx} = \left(\frac{\partial y}{\partial x}\right)_x + \left(\frac{\partial^2 y}{\partial x^2}\right)dx + \left(\frac{\partial^3 y}{\partial x^3}\right)\frac{(dx)^2}{2!} + \cdots\ldots\ldots$$

Considering only the first two terms in the above series

$$\left(\frac{\partial y}{\partial x}\right)_{x+dx} = \left(\frac{\partial y}{\partial x}\right)_x + \left(\frac{\partial^2 y}{\partial x^2}\right)dx \qquad (2)$$

From equations 1 and 2

$$F_y = T\left[\left(\frac{\partial y}{\partial x}\right)_x + \left(\frac{\partial^2 y}{\partial x^2}\right)dx - \left(\frac{\partial y}{\partial x}\right)_x\right] = T\left(\frac{\partial^2 y}{\partial x^2}\right)dx$$

$$\boldsymbol{F_y = T\left(\frac{\partial^2 y}{\partial x^2}\right)dx} \qquad (3)$$

If the mass of the wire per unit length is m, then the mass of the differential element is $m\,dx$.
According to Newton's second law

$$F_y = \text{Mass} \times \text{Acceleration} = (m\,dx)\left(\frac{\partial^2 y}{\partial t^2}\right)$$

$$\boldsymbol{F_y = (m\,dx)\left(\frac{\partial^2 y}{\partial t^2}\right)} \quad \text{——— (4)}$$

From equations 3 and 4

$$(m\,dx)\left(\frac{\partial^2 y}{\partial t^2}\right) = T\left(\frac{\partial^2 y}{\partial x^2}\right)dx$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{m}\left(\frac{\partial^2 y}{\partial x^2}\right)$$

General wave equation is

$$\frac{\partial^2 y}{\partial t^2} = v^2\left(\frac{\partial^2 y}{\partial x^2}\right)$$

$$\therefore v^2 = \frac{T}{m}$$

$$\boldsymbol{v = \sqrt{\frac{T}{m}}}$$

The above equation represents the velocity of transverse wave along a stretched string.
From the above equation it is clear that velocity of transverse wave is

- ✓ Directly proportional to the square root of the tension in the string
- ✓ Inversely proportional to the square root of the linear density of the string.

Modes of vibration of a stretched string clamped at both ends

Consider a string of length l under a tension T clamped at both ends. Let m be the linear density of the string.
General solution of the transverse wave equation of string is given by

$$y = a_1 \sin(\omega t - kx) + a_2 \sin(\omega t + kx) + b_1 \cos(\omega t - kx) + b_2 \cos(\omega t + kx) \quad \text{——— (1)}$$

Boundary conditions of string clamped at both ends are given by

$$At\ x = 0,\ y = 0 \quad \text{——— (2)}$$

$$At\ x = l,\ y = 0 \quad \text{——— (3)}$$

From equations 1 and 2

$$0 = a_1 \sin\omega t + a_2 \sin\omega t + b_1 \cos\omega t + b_2 \cos\omega t$$

$$0 = (a_1 + a_2)\sin\omega t + (b_1 + b_2)\cos\omega t$$

$$a_1 + a_2 = 0, \quad b_1 + b_2 = 0$$

$$\boldsymbol{a_2 = -a_1, \quad b_2 = -b_1}$$

Substituting these values in equation 1

$$y = a_1 \sin(\omega t - kx) - a_1 \sin(\omega t + kx) + b_1 \cos(\omega t - kx) - b_1 \cos(\omega t + kx)$$

$$y = a_1[\sin(\omega t - kx) - \sin(\omega t + kx)] + b_1[\cos(\omega t - kx) - \cos(\omega t + kx)]$$

$$y = a_1[-2\cos\omega t \sin kx] + b_1[2\sin\omega t \sin kx] = (-2a_1)\cos\omega t \sin kx + (2b_1)\sin\omega t \sin kx$$

$$Let\ -2a_1 = A, \quad 2b_1 = B$$

$$y = A\cos\omega t \sin kx + B\sin\omega t \sin kx = \sin kx\,(A\cos\omega t + B\sin\omega t)$$

$$y = \sin kx\,(A\cos\omega t + B\sin\omega t) \quad \text{——— (4)}$$

From equations 3 and 4

$$0 = \sin kl\,(A\cos\omega t + B\sin\omega t)$$

$$\sin kl = 0$$

$$kl = n\pi, \quad n = 1,2,3 \ldots.$$

$$k = \frac{n\pi}{l}$$

$$\boldsymbol{k_n = \frac{n\pi}{l}, \quad n = 1,2,3 \ldots.}$$

Angular frequency

$$\omega_n = k_n v$$

$$\omega_n = \frac{\boldsymbol{n\pi}}{\boldsymbol{l}} v$$

Frequency

$$\nu_n = \frac{\omega_n}{2\pi} = \frac{nv}{2l}, \quad n = 1,2,3 \ldots.$$

Since $v = \sqrt{\frac{T}{m}}$

$$\boldsymbol{\nu_n = \frac{n}{2l}\sqrt{\frac{T}{m}}, \quad n = 1, 2, 3 \ldots.}$$

$If\ n = 1$

$$\boldsymbol{\nu_1 = \frac{1}{2l}\sqrt{\frac{T}{m}}}$$

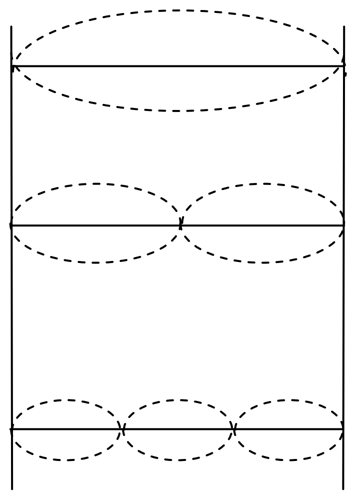
This is known as fundamental frequency or first harmonic.

$If\ n = 2$

$$\boldsymbol{\nu_2} = \frac{\boldsymbol{2}}{\boldsymbol{2l}}\sqrt{\frac{T}{m}} = \boldsymbol{2\nu_1}$$ =Second harmonic or first overtone.

$If\ n = 3$

$$\boldsymbol{\nu_3} = \frac{\boldsymbol{3}}{\boldsymbol{2l}}\sqrt{\frac{T}{m}} = \boldsymbol{3\nu_1}$$ = Third harmonic or second overtone.



Laws of transverse vibration of stretched strings

Fundamental frequency of transverse vibration in a stretched string is given by

$$\boldsymbol{\nu = \frac{1}{2l}\sqrt{\frac{T}{m}}}$$

Here l =Length of the string, T =Tension, m =Linear density

Laws of transverse vibration

1. The fundamental frequency of the string is inversely proportional to the length of the string when T, m are constant.

$$\nu \propto \frac{1}{l}$$

2. The fundamental frequency of the string is directly proportional to the square root of the tension in the string when l, m are constant.

$$\nu \propto \sqrt{T}$$

3. The fundamental frequency of the string is inversely proportional to the square root of the linear density of the string when l, T are constant.

$$\nu \propto \frac{1}{\sqrt{m}}$$

Unit-V

8. ULTRASONICS

Sound waves having frequencies greater than 20000 Hz are called Ultrasonics.
Ultrasonics can be produced using the following two methods.

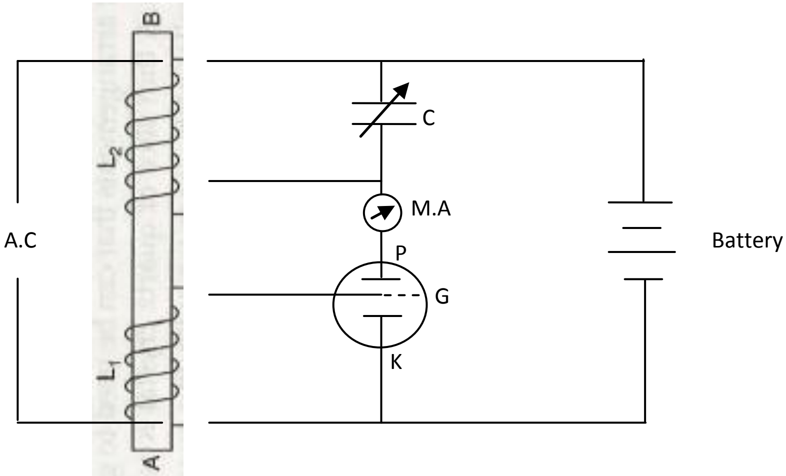
1. Magnetostriction method
2. Piezo-electric Method

Magnetostriction Method

Magnetostriction: When a Ferromagnetic rod is placed in an external magnetic field, it expands slightly in the direction of magnetic field. This phenomenon is called Magnetostriction.

Method:

Ultrasonics can be produced using Magnetostriction method. Consider a Ferromagnetic rod placed inside a coil carrying alternating current. An alternating magnetic field is produced in the rod due to the alternating current. The rod undergoes expansion and contraction due to the alternating magnetic field. Hence the rod vibrates in the direction of the magnetic field with twice the frequency of the alternating magnetic field. But the frequency of the rod should be equal to the frequency of alternating magnetic field to produce ultrasonics. Hence in addition to the alternating magnetic field, a constant magnetic field is also produced by passing direct current through the second coil.



The circuit arrangement for production of ultrasonics is shown in figure. 'AB' is a ferromagnetic rod. Direct current is passed through the coil L surrounding the rod AB. Alternating current is passed through the coils L_1, L_2 to produce an alternating magnetic field. Coil L_1 is connected to the plate circuit while coil L_2 is connected to tank circuit. Current in the plate circuit is measured with a milli ammeter. Frequency of the plate circuit can be adjusted with the help of variable capacitor C. When the frequency of the plate circuit is equal to the natural frequency of the rod, resonance occurs and ultrasonics are produced.

Velocity of Ultrasonics in the rod is $v = \sqrt{\frac{Y}{\rho}}$

Where Y is the Young's Modulus of the rod and ρ is the density of the rod.

$$\text{Fundamental Frequency } \nu = \frac{1}{2l}\sqrt{\frac{Y}{\rho}}$$

- Magnetostriction method is used in the production of low frequency ultrasonics.

Piezo-Electric Method

When pressure is applied along the mechanical axis of a crystal, potential difference is developed along the electric axis. This phenomenon is called Piezo-Electric effect. Crystals which exhibit this phenomenon are called Piezo-Electric crystals.

Example: Quartz, Tourmaline, Rochelle Salt etc…

Converse of Piezo-electric effect is also true. That means when potential difference is applied along the electric exis, pressure is developed along the mechanical axis. If alternating voltage is applied along the electric axis then the crystal vibrates along the mechanical axis. Hence ultrasonics are produced using the converse of piezo-electric effect.

Structure of Quartz Crystal

Quartz crystal belongs to trigonal system. Quartz crystal is a six sided prism with pyramid shaped ends as shown in figure. The cross section of the crystal is a hexagon. Quartz crystal has three different axes.

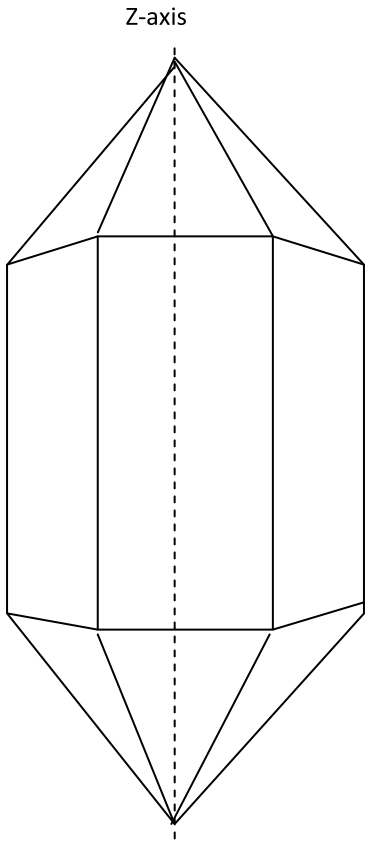
1.Optic axis or Z-axis: The line joining the ends of the pyramids is known as optic axis or Z-axis.

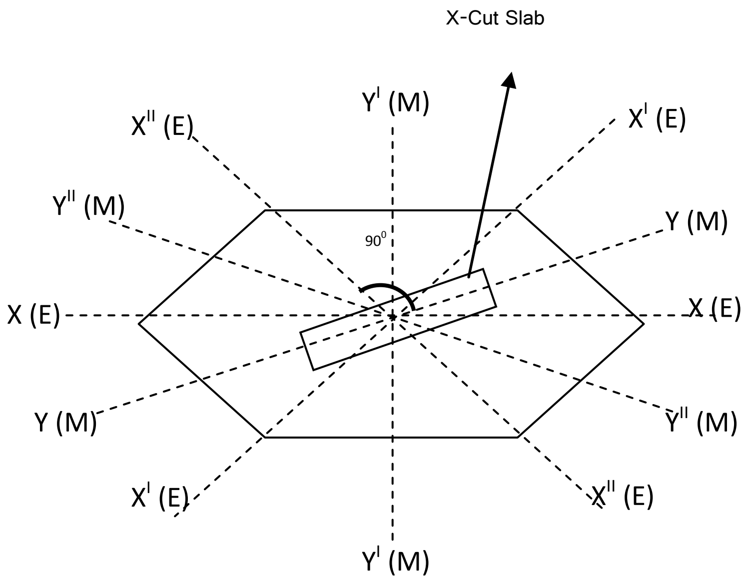
2.Elctric axis or X-axis: The line joining the opposite corners of the hexagon and perpendicular to the optic axis is known as Electric axis.

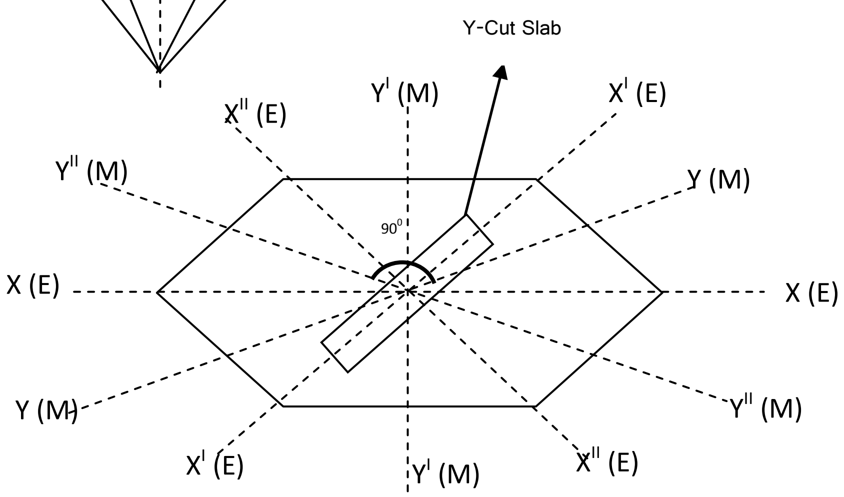
3. Mechanical axis or Y-axis: The line passing through the opposite faces of the hexagon and perpendicular to the optic axis is called the mechanical axis.

X-cut and Y-cut slabs of Quartz crystal are shown in figure.

- X-cut slab makes an angle of 90 with X-axis and Y-cut slab makes an angle of 90 with Y-axis.







Production of Ultrasonics using piezo-electric effect

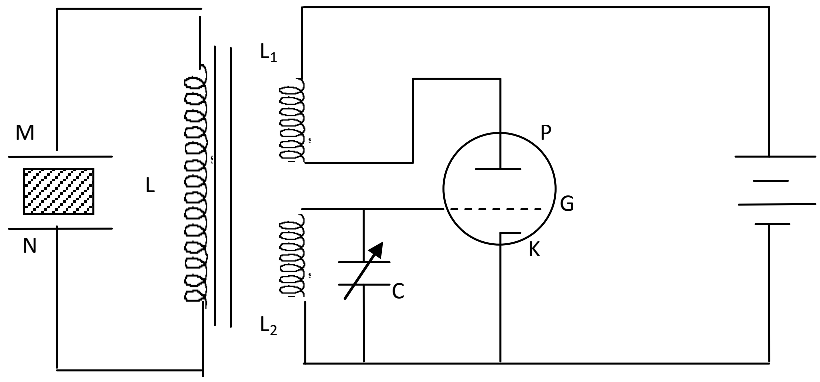
When pressure is applied along the mechanical axis of a crystal, potential difference is developed along the electric axis. This phenomenon is called Piezo-electric effect. Crystals which exhibit this phenomenon are called Piezo-electric crystals.

Example: Quartz, Tourmaline, Rochelle Salt etc…

Converse of Piezo-electric effect is also true. That means when potential difference is applied along the electric exis, pressure is developed along the mechanical axis. If alternating voltage is applied along the electric axis then the crystal vibrates along the mechanical axis. Hence ultrasonics are produced using the converse of piezo-electric effect.

X-cut slab of the crystal is used in the production of ultrasonics since it produces longitudinal waves.

Circuit diagram for the production of ultrasonics using piezo-electric method is shown in figure.



As shown in figure, X-cut slab of the Quartz crystal is placed between two metal plates M and N. These metal plates are connected to the coil L. Coil L_1 is connected to the plate circuit while coil L_2 is connected to tank circuit. Frequency of the plate circuit can be adjusted with the help of a variable capacitor C. When the frequency of plate circuit is equal to the natural frequency of the rod, resonance occurs and ultrasonics are produced.

When the crystal is vibrating with its natural frequency.

$$\lambda = 2t,$$

$$\therefore t = \lambda/2$$

Here t is the thickness of the crystal and λ is the wavelength of ultrasonics.

Velocity of Ultrasonics is

$$V = \sqrt{\frac{Y}{\rho}}$$

Here Y is the Young's Modulus and ρ is the density.

If $v\ is\ the\ frequency\ of\ ultrasonics$, then

$$V = v\,\lambda = v(2t)$$

$$v = \frac{V}{2t} = \frac{1}{2t}\sqrt{\frac{Y}{\rho}}$$

$$v = \frac{1}{2t}\sqrt{\frac{Y}{\rho}}$$

Frequency of the tank circuit $v_T = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$

Detection of Ultrasonics

1.Piezo-eletcric detector:

Ultrasonics can be detected using piezo-electric crystals. When ultrasonics are incident on the quartz crystal along the mechanical axis, alternating voltage is developed along the electric axis. This voltage is amplified to detect ultrasonics.

2.Kund's Tube:

Ultrasonics can be detected using Kund's tube. One end of this tube is fitted with a piston. Lycopodium powder is sprinkled inside the Kund's tube. When ultrasonics are passed through the second end of the tube, stationary waves are produced inside the tube. Hence Lycopodium powder is collected in the form of heaps at nodes.

3.Sensitive Flame method:

When stationary waves of ultrasonics are formed, pressure remains constant at nodes and changes at anti nodes. Hence if a sensitive flame is moved through the region of ultrasonics, the flame remains stationary at nodes and fluctuates at anti nodes.

4.Thermal detector Method:

When stationary waves of ultrasonics are formed, temperature remains constant at nodes and changes at anti nodes. Hence, if a platinum wire is moved through the region of ultrasonics, resistance of the changes at anti nodes and remains constant at nodes.

Applications of Ultrasonics:

1 1. **Communication**:

2 Wavelength of Ultrasonics is very small and have more energy than audible sound. Hence Ultrasonics can be used for communication.

3 2. Determining the depth of Oceans:

4 When high frequency Ultrasonics are pa